AFRICAN ECONOMIC RESEARCH CONSORTIUM

Collaborative Masters Programme in Economics for Anglophone Africa (Except Nigeria)

JOINT FACILITY FOR ELECTIVES (JFE) 2009

JUNE – OCTOBER

ECONOMETRICS THEORY AND PRACTICE II

Second Semester: Final Examination

Duration: 3 Hours **Date: Thursday, October 1, 2009**

INSTRUCTIONS:

Answer **ANY THREE (3)** Questions.

All questions carry equal weight. Credit will be given for orderly presentation of **relevant** materials

Question 1

The following is a panel of data on investment (y) and profit (x) for n=3 firms over T=10 periods.

	i=1		i=2		i=3	
Time	Y	X	Y	X	Y	X
t= 1	13.32	12.85	20.30	22.93	8.85	8.65
t= 2	26.30	25.69	17.47	17.96	19.60	16.55
t= 3	2.62	5.48	9.31	9.16	3.87	1.47
t= 4	14.94	13.79	18.01	18.73	24.19	24.91
t= 5	15.80	15.41	7.63	11.31	3.99	5.01
t= 6	12.20	12.59	19.84	21.15	5.73	8.34
t= 7	14.93	16.64	13.76	16.13	26.68	22.70
t= 8	29.82	26.45	10.00	11.61	11.49	8.36
t= 9	20.32	19.64	19.51	19.55	18.49	15.44
t=10	4.77	5.43	18.32	17.06	20.84	17.87

(a) Pool the data and compute the least squares regression coefficients of the model
$$y_{it} = \alpha + \beta' x_{it} + \varepsilon_{it}.$$ **[8 marks]**

(b) Estimate the fixed effects model, then test the hypothesis that the constant term is the same for all three firms. **[9 marks]**

(c) Estimate the random effects model, then carry out the Lagrange multiplier test of the hypothesis that the classical model without the common effect applies. **[8 marks]**

(d) Carry out Hausman's specification test for the random versus the fixed model. **[8 marks]**

Question 2

The Belgian government is contemplating increasing the tax on tobacco in order to lower the incidence of smoking. A consultant to the government therefore wishes to examine whether such a tax increase will actually lower the demand for tobacco. He knows that in order to assess this he needs to take both the price and income effect into account. He has an estimate of the price elasticity for tobacco and he turns to you for assistance in estimating the income elasticity. The data he has available for estimating the income elasticity is the Belgian Household Budget Survey 1995-1996, which contains information on household, how many adults live in the household as well as the age class of the head of the household for $n = 2724$ households. He has estimated a linear regression model on this data by running OLS of the budget share for tobacco (*btobacco*) on log total expenditure (*lnx*), number of children in the household (*nkids, nkids2*), number of adults in the household (*nadults*) and age (*age*). The Stata output for this regression is given in appendix A.

Denoting the budget share of tobacco by ω and the log total expenditure by ln x, the income elasticity e for tobacco can be calculated by the formula

$$e = \frac{1}{\omega}\frac{\partial \omega}{\partial lnx} + 1$$

(a) Calculate the income elasticity e_{OLS} resulting from the OLS. **[6 marks]**

(b) How would you (briefly) explain to the consultant that the linear regression model my not be appropriate model for explaining tobacco expenditures for this sample (no mathematical proof is needed) **[6 marks]**

(c) Given the following data set:

Y | 1 0 0 1 1 0 0 1 1 1

X | 9 2 5 4 6 7 3 5 2 6

(i) Estimate a probit model **[6 marks]**

(ii) Test the hypothesis that X is not influential in determining the probability that Y equals one. **[5 marks]**

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Question 3

Suppose we are interested in explaining wages. Also suppose we have data on education levels, years of labour market experience and ages for n individuals, as well as the wages for those individuals that work.

(a) Explain why estimating a linear regression of wages on education levels, years of labour market experience and ages by OLS may not be appropriate. **[12 marks]**

(b) We are interested in the ordered probit model. Our data consist of 250 observations, of which the responses are

Y	0	1	2	3	4
n	50	40	45	80	35

Using the data above, obtain maximum likelihood estimates of the unknown parameters of the model. [Hint: Consider the probabilities as the unknown parameters] **[10 marks]**

(c) (i) Construct the Lagrange multiplier statistic for testing the hypothesis that all of the slopes (but not the constant term) equal zero in the binomial logit model. **[5.5 marks]**

(ii) Prove that the Lagrange multiplier statistic is nR^2 in the regression of $(y_i - P)$ on the xs, where P is the sample proportion of ones. **[5.5 marks]**

Question 4

The following 20 observations are drawn from a censored normal distribution:

3.8396, 7.2040, .00000, .00000, 4.4132, 8.0230, 5.7971, 7.0828, .00000, .80260, 13.0670, 4.3211, .00000, 8.6801, 5.4571, .00000, 8.1021, .00000, 1.2526, 5.6015.

The applicable model is

$y_i^* = \mu + \varepsilon_i$
$y_i = y_i^*$ if $\mu + \varepsilon_i > 0$, 0 otherwise.
$\varepsilon_i \sim N[0,\sigma2]$.

The OLS estimator of μ in the context of this tobit model is simply the sample mean.

(a) Compute the mean of all 20 observations. **[6 marks]**

(b) Would you expect this estimator to over- or underestimate μ? If we consider only the nonzero observations, the truncated regression model applies. **[6 marks]**

(c) The sample mean of the nonlimit observations is the least squares estimator in this context. Compute it, then comment on whether this should be an overestimate or an underestimate of the true mean. **[10 marks]**

(d) Refer to the data above. We now consider the tobit model that applies to the full data set.

(i) Formulate the log-likelihood for this very simple tobit model. **[6 marks]**

(ii) Reformulate the log-likelihood in terms of $\theta = 1/\sigma$ and $\gamma = \mu/\sigma$. Then, derive the necessary conditions for maximizing the log-likelihood with respect to θ and γ. **[5 marks]**

Question 5

(a) Explain the following duration concepts: **[6marks]**

- (i) Right censoring and left censoring
- (ii) Failure function
- (iii) Baseline hazard function

(b) Derive the survivor rate function. What is the relationship between the specification of the hazard rate and the survivor rate functions? [Mathematical treatment is necessary] **[10 marks]**

(c) Given the basic Cox proportional hazard model:

$$\lambda(t|x_i) = \lambda_0(t)exp(\beta_1 X_1 + \beta_2 X_2 +, \dots, +\beta_k X_k)$$

- (i) Why is this model referred to as a semi parametric model? **[4 marks]**
- (ii) Discuss the method of estimation of the parameters $\beta_1, \beta_2, \dots, \beta_k$. **[4 marks]**

(d) Suppose the Weibull model hazard rate is given by

$$\lambda(t, x) = \alpha^{t-1} exp(x'\beta)$$

- (i) Under what conditions do all the results of the Weibull model apply to the exponential model? **[3 marks]**
- (ii) Assuming x_k is the Kth covariate in the vectors of characteristics X for two persons i and j. Show how each coefficient summarises the proportionate response of the hazard of the absolute changes in the relevant covariate. **[6 marks]**

Appendix A

```
. reg btobacco lnx age nadults nkids2 nkids

      Source |       SS       df       MS              Number of obs =    2724
-------------+------------------------------           F(  5,  2718) =   40.32
       Model |  .116758246     5  .023351649           Prob > F      =  0.0000
    Residual |   1.5741102  2718  .000579143           R-squared     =  0.0691
-------------+------------------------------           Adj R-squared =  0.0673
       Total |  1.69086845  2723  .000620958           Root MSE      =  .02407

------------------------------------------------------------------------------
    btobacco |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         lnx |  -.0141745   .0011452   -12.38   0.000    -.0164202   -.0119289
         age |  -.0025072   .0003865    -6.49   0.000    -.0032651   -.0017493
     nadults |   .0027508   .0006524     4.22   0.000     .0014716    .0040301
      nkids2 |  -.0047776   .0022332    -2.14   0.032    -.0091565   -.0003987
       nkids |    .001168   .0005623     2.08   0.038     .0000654    .0022705
       _cons |   .2069766   .0151311    13.68   0.000     .1773069    .2366462
------------------------------------------------------------------------------

. predict wreg, xb

. summarize wreg

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
        wreg |      2724    .0321908    .0062637   .00128599    .060071
```

```
. tab d2

 dummy=1 if |
    tobacco |
expenditure |
         >0 |      Freq.     Percent        Cum.
------------+-----------------------------------
          0 |      1,688       61.97       61.97
          1 |      1,036       38.03      100.00
------------+-----------------------------------
      Total |      2,724      100.00
```

```
. tobit btobacco lnx age nadults nkids2 nkids , ll(0)

Tobit regression                                  Number of obs   =       2724
                                                  LR chi2(5)      =     145.58
                                                  Prob > chi2     =     0.0000
Log likelihood =  746.40082                       Pseudo R2       =    -0.1081

------------------------------------------------------------------------------
    btobacco |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         lnx |  -.0256124   .0027221    -9.41   0.000       -.03095   -.0202748
         age |   -.006387   .0009186    -6.95   0.000     -.0081882   -.0045858
     nadults |   .0076941    .001545     4.98   0.000      .0046645    .0107237
      nkids2 |  -.0135256   .0054335    -2.49   0.013     -.0241798   -.0028714
       nkids |   .0029758   .0012966     2.29   0.022      .0004333    .0055183
       _cons |    .334203   .0357935     9.34   0.000      .2640178    .4043883
-------------+----------------------------------------------------------------
      /sigma |   .0483493   .0011926                      .0460108    .0506879
------------------------------------------------------------------------------
  Obs. summary:       1688  left-censored observations at btobacco<=0
                      1036     uncensored observations
                         0 right-censored observations

. predict wtobit, e(0,1)

. summarize wtobit

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
      wtobit |      2724    .0334894    .0040911   .0230732   .0575392
```